Answers to examination-style questions

Answers	Marks	Examiner's tips
1 (a) $v_{\text{esc}} = \sqrt{2gR}$	**1**	
$= (2 \times 1.62 \times 1740 \times 10^3)^{½}$		
$= 2400 \text{ m s}^{-1}$	**1**	
(2370 m s^{-1} to 3 sig figs)		
(b) (i) Mean kinetic energy $= \frac{3}{2}kT$		
$= 1.5 \times 1.38 \times 10^{-23} \times 400$	**1**	
$= 8.3 \times 10^{-21}$ J	**1**	
(ii) Mass of an oxygen molecule m = molar mass/N_A	**1**	
$= \dfrac{0.032}{6.02 \times 10^{23}}$ kg		
Rearranging $\frac{1}{2}mc_{\text{rms}}{}^2 = 8.3 \times 10^{-21}$ J …	**1**	
gives $c_{\text{rms}}{}^2 = \dfrac{2 \times 8.3 \times 10^{-21}}{0.032/6.02 \times 10^{23}}$	**1**	
$= 560 \text{ m s}^{-1}$		
(iii) Gas molecules have a range of speeds (or kinetic energies)	**1**	
• The escape speed from the lunar surface is about 4 × the rms speed of the gas molecules • Some gas molecules would have sufficient kinetic energy (or speed) to escape from the lunar surface • Each day more gas molecules would escape • As they would gain kinetic energy from the hot surface	**any 2**	
(c) (i) • Absorption lines are due to the absorption of light of certain wavelengths • The lines were observed only when the planet passed across the face of the star • If the star had caused the absorption lines, they would have been present all the time (∴ the planet must have absorbed light at these wavelengths)	**6 marks max**	

Answers	Marks	Examiner's tips
(ii) Either: • there would be less absorption by a smaller planet • as it would cover a smaller % of the star's surface when it is in front of the star • so the absorption lines would be harder to see. or: • it might not have retained an atmosphere • because the escape velocity of molecules from a smaller planet would be smaller • the mean kinetic energy of the molecules (in its atmosphere) would be the same.		
2 (a) *Graph on axes labelled pressure and volume to show*: **(i)** a correct $p \propto \frac{1}{V}$ curve labelled **O**	1	When the pressure is high, the volume is small; when the pressure is low the volume is large. Your graphs should be curves (none of which meet either axis) of decreasing negative gradient, of the form $y \propto \frac{1}{x}$. At a lower temperature all pressures on **A** will be lower than **O** at the same volume values. A greater mass of gas will cause the pressures on **B** to be higher than **O** at the same volume values.
(ii) a curve labelled **A** that is the same shape as but lower than **O** at all values	1	
(iii) a curve labelled **B** that is the same shape as but higher than **O** at all values.	1	
(b) (i) $pV = nRT$ $\therefore\ 130 \times 10^3 \times 0.20 = n \times 8.31 \times 290$ . . . from which amount of gas $n = 10.8$ mol	1 1	Remember to substitute the value of pressure in Pa, not kPa. When using this equation the temperature must be in K.
(ii) Average kinetic energy of a molecule $= \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 290$ $= 6.00 \times 10^{-21}$ J	1 1	The result $\frac{1}{2}mc_{\text{rms}}{}^2 = \frac{3}{2}kT$ follows from the kinetic theory. On average, each molecule has an amount of kinetic energy $\frac{3}{2}kT$, where k is the Boltzmann constant. Hence the total kinetic energy of all the molecules is found by multiplying the value found in part **(ii)** by the number of molecules in the cylinder.
(iii) Number of molecules in cylinder $= 10.8 \times 6.02 \times 10^{23} = 6.50 \times 10^{24}$ $\therefore$ total kinetic energy of the molecules $= 6.50 \times 10^{24} \times 6.00 \times 10^{21}$ $= 3.90 \times 10^4$ J	1	

Answers	Marks	Examiner's tips
3 (a) The equation of state for an ideal gas is $pV = nRT$ (where p = pressure, V = volume, T = absolute temperature, n = amount of gas in mol, and R = molar gas constant).	**1**	This equation defines the behaviour of what is known as an ideal gas. It is sometimes called the ideal gas equation. The equation of state could equally be written as $pV = NkT$ (for N molecules), where k is the Boltzmann constant.
(b) *Graph on axes labelled pressure and temperature to show:* • a straight line of positive gradient • which passes through an origin marked (0, 0).	**2**	For a fixed mass of ideal gas, the pressure is proportional to the absolute temperature. The experimental relationship between these quantities is the pressure law. Direct proportion between two quantities is always shown by a straight line graph passing through the origin.
(c) *Relevant points include the following listed below.* • When the temperature is increased the average kinetic energy of the molecules (or their speed) increases • There are more collisions per second against the walls containing the gas • The average change in momentum in each collision is greater … • leading to a greater rate of change of momentum • Force = rate of change of momentum, and pressure = $\frac{\text{force}}{\text{area}}$ so there is a greater pressure on the walls.	**any 4**	At a higher temperature the molecules travel faster on average, taking less time to move across a container. Therefore collisions with the walls are more frequent. Each collision involves a molecule with greater average momentum, producing a larger impulse per collision. So there are more collisions per second, and each collision causes a larger force on the wall. Hence the pressure exerted by the gas increases.
(d) Average kinetic energy of a molecule $= \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$	**1**	Always read the wording of a question on this topic very carefully. $\frac{3}{2}kT$ is the average kinetic energy of one molecule of the gas. Sometimes a question similar to this may intend you to calculate the total kinetic energy of all of the molecules in the gas.
$= 6.21 \times 10^{-21}$ J	**1**	
4 (a) p = pressure and V = volume	**1**	You are required to be familiar with the derivation of $pV = \frac{1}{3}Nmc_{\text{rms}}^2$, which involves at least knowing what each of the symbols represents!
N = number of molecules	**1**	
m = mass of one molecule (or particle)	**1**	
c_{rms}^2 = mean square speed of molecules	**1**	

Answers	Marks	Examiner's tips
(b) **(i)** Molecules have a range of speeds … and no preferred direction of motion.	**1** **1**	There are two aspects to the random motion of the molecules in a gas. One is that there is a great variation in the speeds of individual molecules, which will change in molecular collisions. The other is that they can move in any direction. Their directions of movement will also be changed frequently by molecular collisions.
(ii) *Other possible assumptions*: • the collisions are perfectly elastic • intermolecular forces are negligible (except during collisions) • the volume of the molecules is negligible (compared to volume of container) • the duration of collisions is negligible (compared to the time between collisions) • the molecules are identical • a gas consists of a very large number of molecules • the molecules behave according to the laws of Newtonian mechanics.	**any 2**	It should not be difficult to recall two assumptions from such a long list. Take care that you do not state assumptions that mean the same thing as one other. For example, molecules travel in straight lines between collisions because the forces between them are negligible; therefore these are not two independent assumptions.
5 **(a)** **(i)** At sea level: $pV = nRT$ gives $1.0 \times 10^5 \times 1.0 = n \times 8.31 \times 300$ from which $n = 40.1$ mol	**1** **1**	Because these calculations are repetitive, more credit is given for getting the first one correct than the second one. In each case the volume to be considered is one cubic metre (= 1.0 m^3).
(ii) At 10 000 m: $pV = nRT$ gives $2.2 \times 10^4 \times 1.0 = n \times 8.31 \times 270$ from which $n = 9.81$ mol	**1**	
(b) Total number of gas molecules in 1.0 m^3 of air at sea level: $= 40.1 \times 6.02 \times 10^{23}$ $= 2.41 \times 10^{25}$ and at 10 000 m: $= 9.81 \times 6.02 \times 10^{23} = 5.91 \times 10^{24}$ $\therefore$ extra number of molecules at sea level: $= (2.41 \times 10^{25}) - (5.91 \times 10^{24})$ $= 1.82 \times 10^{25}$ *Number of extra oxygen molecules* $= 0.23 \times 1.82 \times 10^{25} = 4.19 \times 10^{24}$	**1** **1**	You have to think clearly before you embark on this part. Your calculations in part **(a)** mean that you can easily work out the total number of molecules in 1.0 m^3 of air at sea level, and at 10 000 m. Therefore you can find the number of extra molecules at sea level, and you know that 23% of them are oxygen molecules. 10 000 m is a typical altitude of a jet aircraft in flight. It is because the air is so 'thin' at these heights that jet aircraft have pressurised cabins to enable passengers to breathe normally.

	Answers	Marks	Examiner's tips
6 (a) (i)	Change in momentum of molecule in an elastic collision $= mv - (-mv) = 2\,mv$ $= 2 \times 6.8 \times 10^{-27} \times 1300$ $= 1.77 \times 10^{-23}$ N s (or kg m s^{-1})	1 1	Momentum is a vector quantity. If the molecule were brought to rest, the change in momentum would be mv. In this elastic collision the velocity is exactly reversed, so the change in momentum is twice what it would be if it were brought to rest.
(ii)	Force required on 1.0 m^2 = 1.5×10^5 N $\therefore$ force required on 1.0 cm^2 $= \dfrac{1.5 \times 10^5}{100^2} = 15$ N Force = rate of change of momentum $\therefore\ 15 = \dfrac{\Delta N}{\Delta t} \times 1.77 \times 10^{-23}$ $\therefore$ number of collisions per second on 1 cm^2, $\dfrac{\Delta N}{\Delta t} = 8.47 \times 10^{23}$ s^{-1}	1 1 1	1 Pa = 1 N m^{-2}; the force required on 1.0 cm^2 is 10 000 times less than that on 1.0 m^2. Each collision causes a change in momentum of 1.77×10^{-23} N s, so the number of collisions required per second is found by dividing the force required on 1.0 cm^2 by the change in momentum in one collision.
(b) (i)	Average kinetic energy of a molecule $= \tfrac{3}{2}kT$ $\therefore\ 8.6 \times 10^{-21} = \tfrac{3}{2} \times 1.38 \times 10^{-23} \times T$ … from which temperature of sample $T = 415$ K	1 1	In this instance the question requires you to use $\tfrac{3}{2}kT$ in order to find a temperature.
(ii)	Mass of a nitrogen molecule $m = \dfrac{0.028}{6.02 \times 10^{23}} = 4.65 \times 10^{-26}$ kg Average kinetic energy of a molecule $= \tfrac{1}{2}mc_{\mathrm{rms}}{}^2$ $\therefore\ 8.6 \times 10^{-21} = \tfrac{1}{2} \times 4.65 \times 10^{-26} \times c_{\mathrm{rms}}{}^2$ gives mean square speed of nitrogen molecules $c_{\mathrm{rms}}{}^2 = 3.70 \times 10^5$ m^2 s^{-2}	1 1 1	The molar mass (0.028 kg mol^{-1}) contains N_A (6.02×10^{23}) molecules. Be careful to write the average kinetic energy of a molecule as $\tfrac{1}{2}mc_{\mathrm{rms}}{}^2$, and not as $\tfrac{1}{2}mv^2$; the symbol v in the latter gives no indication of an average value. The unit of the final answer also requires care: it is the unit of (speed)^2.
7 (a)	Gradient of graph $= \dfrac{14.0 - 11.2}{100 - 20} = 0.035$ cm^3 °C^{-1} intercept on volume axis at 0 °C = 10.5 cm^3 gradient $= \dfrac{10.5}{\theta_0} = 0.035$ gives $\theta_0 = 300$ °C $\therefore$ absolute zero of temperature $= -300$ ($\pm$ 10) °C	1 1 1 1	Absolute zero is the temperature at which the volume of an ideal gas would shrink to nothing. It can be found from the graph of V against θ by extrapolating the line to the temperature at which $V = 0$. This is often done most accurately by calculation from the value of the gradient. Similar triangles are the basis of the calculation: over any section of the same straight line, $\dfrac{\text{change in } V}{\text{change in } \theta} = \text{constant} = \text{gradient}.$

Answers	Marks	Examiner's tips
(b) Taking any pair of values from the line e.g. $V = 14.0\ \text{cm}^3$ when $\theta = 100\ °\text{C}$ converting to absolute temperature: $100\ °\text{C} = 100 + 273 = 373\ \text{K}$ and substituting values into $pV = nRT$ gives $1.0 \times 10^5 \times 14.0 \times 10^{-6} = n \times 8.31 \times 373$ from which $n = 4.52 \times 10^{-4}$ mol and mass of gas used in experiment $= 4.52 \times 10^{-4} \times 0.044 = 1.99 \times 10^{-5}$ kg	1 1 1 1 1	To find the amount of gas in moles, you need a pair of values from the graph for V and θ. It would be permissible to use your answer from part **(a)**, for example 300, instead of 273 when changing θ/°C into T/K. One mole of the gas has mass 0.044 kg, so the mass of gas in the experiment was $n \times 0.044$ kg.
(c) *Relevant points include the ones listed below.* In either part: • pressure is caused by the collisions of the molecules against the container • pressure is the change in momentum of the molecules per unit area per second. In **(i)** at constant temperature: • when volume increases the molecules have further to travel between collisions • there are fewer collisions per second against the container walls. In **(ii)** at constant volume: • when the temperature is reduced the molecules have a lower mean speed (or kinetic energy) • the change in momentum (or impulse) per collision is reduced. • there are fewer collisions per second with the walls	**any 5**	Essentially, the kinetic theory explains that the pressure of a gas is caused by collisions of its molecules against the walls of a container. Each collision causes a change in the momentum of a molecule, resulting in an impulse on the wall. Force is rate of change of momentum, and pressure is force per unit area. The kinetic theory also shows that the absolute temperature is proportional to the mean molecular kinetic energy. If the temperature falls, the molecules will move more slowly on average, making fewer collisions per second and causing a smaller impulse on average in each collision.
8 **(a)** **(i)** $pV = nRT$ $\therefore\ 500 \times 10^3 \times V = 15 \times 8.31 \times 290$ gives $V = 7.23 \times 10^{-2}\ \text{m}^3$	1 1	In this kind of question it is generally best to quote your final answer to one more significant figure than the 7.2×10^{-3} given in the question. This shows that you have worked out the value of V, and not simply re-written the value given in the question.
(ii) Average kinetic energy of a gas molecule $= \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 290$ $= 6.00 \times 10^{-21}$ J	1 1	Past examination questions give plenty of practice on this topic.
(b) $pV = nRT$ $\therefore\ 420 \times 10^3 \times 7.23 \times 10^{-2} = n \times 8.31 \times 290$ gives amount of gas remaining in cylinder $n = 12.6$ mol	1 1	The pressure now falls to 420 kPa but the volume and temperature remain the same as they were in part **(a)**.

Answers	Marks	Examiner's tips
(c) *Relevant points include*: • pressure is caused by the collisions of the molecules against the container • pressure is the change in momentum of the molecules per unit area per second • when gas is removed there are fewer molecules in the cylinder • there are fewer collisions per second against the container walls • the impulse per collision is, on average, the same as it was originally • since c_{rms}^2 is constant (because the temperature is unchanged).	**any 4**	*Alternative (more mathematical) approach*: • using $pV = \frac{1}{3}Nmc_{rms}^2$ • in this example V and m are constant • c_{rms}^2 is constant because the temperature is unchanged • therefore $p \propto N$ • when gas is removed, N decreases and so p decreases.